

Determination of transmission reliability margin considering uncertainties of system operating condition and transmission line outage

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SUMMARY

The uncertainty of system operating conditions is a part of consequences which may cause to the volatility of a transmission system. This will hinder the performance of transmission system to effectively transfer the power between areas. Therefore, accurate estimation of transmission reliability margin (TRM) is required to ensure effective power transfer between areas during the occurrence of uncertainties. The power transfer is also called as the available transfer capability (ATC) in which it is the information required by the utilities and marketers to instigate selling and buying the electric energy. The TRM is estimated by taking into account the uncertainties of line outages and system parameters generated by the bootstrap technique. A case study of Malaysia Power System is used to verify the robustness of bootstrap technique in the TRM determination. The results show that the combined impact of several uncertainties which significantly affect the value of TRM. Copyright © 2010 John Wiley & Sons, Ltd.

KEY WORDS: available transfer capability; transmission reliability margin; bootstrap technique; line outage; system parameters

1. INTRODUCTION

Deregulation of electric power industry is to allow competition among generation and distribution companies in order to create market condition in the electrical utilities; and also to increase the quality and efficiency of electric energy production that can offer lower price of electricity. In a more competitive electric power market, transmission providers are required to produce commercially viable information of available transfer capability (ATC) so that such information can help power marketers, sellers and buyers in planning, operation and reserving transmission services. ATC is defined as the additional amount of power that may flow across the interface, over and above the base case flows and it indicates as the amount by which interarea power transfers can be increased without jeopardizing system security. Mathematically, ATC is defined as the total transfer capability (TTC) less the transmission reliability margin (TRM), less the capacity benefit margin (CBM) and less the base case power flow [1,2]. By definition, TTC represents the maximum amount of power that can be transferred over a transmission network while meeting all of the specific set of defined pre- and post-contingency system conditions. CBM is defined as the amount of transmission capability reserved by load serving entities to meet the generation reliability requirements, while TRM is the amount of transmission capability necessary to ensure that the transmission network is secure under a reasonable range of uncertainty in system operating condition. There are various methods in calculating the TTC and ATC [3–5]. In the deregulated environment, one is more concern on the TTC less the base case

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power flow (or ATC) instead of TTC [3]. Therefore, the power transfer mentioned in this paper is referring to the ATC and it is also considered in the TRM determination.

This paper presents a new method that used to estimate the TRM by considering the uncertainties of transmission line outages and system parameters determined by using the bootstrap technique. In particular, the parametric bootstrap technique is used to generate the uncertainties of system parameters which are transmission line impedance and hourly peak loads in a day. Then, the ATCs in a day are calculated at two cases which are the uncertainties of transmission line impedances and hourly peak loads in a day. It is worth mentioning that several samples of transmission line outages are determined by using the bootstrap technique. Then, the ATC is also calculated for every sample of transmission line outages. Simultaneously, the ATCs are used in the computation of TRM at every time interval in a day. The actual TRM value is obtained by amalgamating all the TRM values determined by considering the uncertainties of transmission line impedance, transmission line outages and hourly peak loads. The TRM value at a particular time interval is selected based on a certain percentage of normal cumulative distribution function (CDF). The percentage of normal CDF is also called as the percentile of variability. Variability represents diversity or heterogeneity in data, which is not reducible through further measurement or study [6–8]. On the other hand, uncertainty arises due to the limited amount of information that is used to characterize the entire data [6–8] and it is used in estimating the TRM. Then, a new value of ATC at current time interval is calculated by considering the actual TRM value at the same time interval. The effectiveness of the proposed TRM method in determining the ATC is validated on the Malaysian power system. The results have shown that the uncertainty of transmission line outages significantly affects the value of TRM. The advantage of using the parametric bootstrap technique is that accurate TRM value can be obtained that considers large uncertainty of ATC. The other advantage is that the parametric bootstrap technique provides realistic information of uncertainty in the TRM determination. The uncertainty is said to be realistic because the ATCs provided by the parametric bootstrap technique are not constrained between the maximum and minimum range of inherent or actual variables of ATC [9], and this may occur in the future condition of transfer capability. On the other hand, disadvantage of using the proposed method is that it provides a lengthy total computational time in determining the TRM due to the resampling process involved in parametric bootstrap technique. However, prior to the advance development in computer technology, this may not be a predicament in determining the TRM with less computational time.

2. DETERMINATION OF TRM USING BOOTSTRAP TECHNIQUES CONSIDERING THE UNCERTAINTIES OF SYSTEM OPERATING CONDITION

In this section, the implementation of parametric bootstrap technique that used to estimate large uncertainty of ATCs is first described and then followed by the explanation of TRM determination. Generally, the parametric bootstrap technique is used to randomly generate the ATCs with large uncertainty at a certain percentage of bootstrap confidence interval. Then, the TRM is obtained by taking into account the large uncertainty of ATCs.

2.1. Estimation of transfer capability uncertainty using parametric bootstrap technique

In a deregulated power system, greater variations of power generation and network system condition are caused by the growing number of trading activities in a power market. These variations in system operating condition are the information which requires large data storage. However, operation practice does not allow storing a large number of systems operating condition data [10–12]. To overcome the problem, a small number of system operation conditions are pertinent only for data storage and then it is used in the probabilistic method such as the non-parametric bootstrap technique to produce large number of system operating condition with uncertainty. Tsai *et al.* [11] uses the non-parametric bootstrap technique in estimating the uncertainty based on a small sample size of injected power at each bus. Another technique which provides more realistic information of uncertainty is the parametric bootstrap technique that able to predict the parameters with uncertainty beyond the range of observed data. The parametric bootstrap technique is robust in making predictions of parameters with

uncertainty as compared to the non-parametric bootstrap technique which only estimate between the maximum and minimum values of the observed data [9], and this condition may occur in future.

The value of TRM with large uncertainty is obtained by referring to the large uncertainty of ATCs provided by the parametric bootstrap technique. The implementation of parametric bootstrap technique in estimating the large uncertainty of ATCs is described as follows:

- (a) A sample of data points, $ATC_1, ATC_2, ATC_3, \dots, ATC_n$, is assumed to be the inherent variables of ATC in a day for a specified transfer case, where $n = 1, 2, 3, \dots, N$ and N is the total number of time intervals in a day. The inherent variables of system operating condition are used in the first order sensitivity method to determine the inherent variables of ATC and it is given by (1)

$$ATC_n = ATC^0 + \frac{\partial ATC}{\partial x}(x_n - x^0) \quad (1)$$

The x_n is used in (1) to determine the ATCs that are different compared to the base case ATC that is determined by referring to the system operating condition at base case, x^0 . The first order sensitivity $\partial ATC/\partial x$ used in (1) provides fast computation of ATC and it is based on the limiting point of system constraints [13,14]. The sensitivity method provides approximate value of ATC and the error produced by the approximation can be neglected since the uncertainties of system operating condition gives more significant effect on the ATC [15]. Hence, the ATC considering uncertainties of system operating condition can be taken into account in the TRM determination. Audomvongseeree *et al.*[16] and Gravener *et al.*[17] explained that the $\partial ATC/\partial x$ can be calculated directly by solving two recursive AC power solutions. Firstly, the recursive AC power flow is performed by referring to the base case value of system parameter, x^0 . By slightly increasing the system parameter x , then the second recursive AC power flow solution is performed that varies the value of power transfer. Then, the sensitivity is calculated as the change of ATC with respect to the change of system parameter. A detail explanation on the procedure of recursive AC power flow solution is described in Section 4. The highest amount of uncertainty is stipulated by the peak load [18,19]. In the second procedure of executing the recursive AC power flow solution for sensitivity estimation, the increased amount of peak load, x , is distributed to each PQ bus based on the ratio of load at each PQ bus and the total load. Then, the total generation is increased in order to provide sufficient amount of supply required by the demand. Hence, the amount of increased total generation is similar to the amount of increased peak load. The increased total generation is distributed to each PV bus based on the ratio of generation at each PV bus and the total generation. By executing the AC power flow solution, the power mismatch between the increased total generation and increased peak load is compensated by the generation at slack bus. This basic procedure is performed due to the lack of information or data of the generator type, load profile at each PQ bus and fuel cost for each generator as appeared in this case study. Hence, the chronological of load profile for each PQ bus is the same as the chronological of hourly peak loads in a day. The above-mentioned explanation is not a firm procedure used in allocating the power to each PV or PQ buses. Infact, Ou *et al.*[20] uses the procedure that evenly distributes the power to each PV bus.

- (b) Determine the parametric distribution that fits to the empirical distribution function of inherent variables. The best representative of parametric distribution chosen in this analysis is the normal CDF.
- (c) Use a random number generator to draw a random sample of n values, and replace the measured sample data points, ATC_n , with the non-parametric bootstrap sample, ATC_n^* .
- (d) Repeat step (c) in order to obtain a total number, B , of non-parametric bootstrap samples, $ATC_{n,b}^*$, where $b = 1, 2, 3, \dots, B$.
- (e) Determine the mean and variance of each non-parametric bootstrap sample. The mean and variance are the parameters of a normal CDF. The mean of each non-parametric bootstrap sample is given by,

$$\mu(ATC_b^*) = \frac{1}{N} \sum_{n=1}^N ATC_{n,b}^* \quad (2)$$

and the variance of each non-parametric bootstrap sample is given by,

$$\sigma^2(ATC_b^*) = \frac{1}{N-1} \sum_{n=1}^N \left[ATC_{n,b}^* - \mu(ATC_b^*) \right]^2 \quad (3)$$

- (f) Then, the $\mu(ATC_b^*)$ and $\sigma^2(ATC_b^*)$ are used in (4) to obtain the parametric bootstrap samples with normal random variables. The normal random variables represents as the ATCs in a day with large uncertainty and it is given by,

$$ATC_{n,b} = \text{rand}_n \left[\sqrt{\sigma^2(ATC_b^*)} \right] + \mu(ATC_b^*) \quad (4)$$

- (g) Arrange the parametric bootstrap samples, $ATC_{n,b}$ by referring to the $\mu(ATC_b^*)$ sorted in ascending order.
 (h) Then, a parametric bootstrap sample is selected based on the confidence interval of the mean of parametric bootstrap sample. The desired $(1-\alpha)$ 100% bootstrap confidence interval of uncertainty is between the range of $ATC_{n,b=q1}$ and $ATC_{n,b=q2}$, where $q1 = (B\alpha/2)$ and $q2 = B - q1 + 1$. Note that for 95% bootstrap confidence interval, $\alpha = 0.05$. In which, α is a degree of confidence. A parametric bootstrap sample, $ATC_{n,b=q1}$ is then used in the TRM calculation.

The procedure of non-parametric bootstrap is based on steps (a–e), (g) and (h), and it can also be used in the TRM determination. The parametric bootstrap samples, $ATC_{n,b}$ in step (g) are replaced by the non-parametric bootstrap samples, $ATC_{n,b}^*$ obtained in (d). Hence, $ATC_{n,b=q1}$ in step (h) represents as the non-parametric bootstrap sample which is selected based on the confidence interval of the mean of non-parametric bootstrap sample. A non-parametric bootstrap sample, $ATC_{n,b=q1}$, is then used in the TRM calculation.

2.2. TRM determination using bootstrap techniques

The procedure of TRM determination for each case of power transfer using both of the bootstrap techniques is described as follows:

- (a) Establish a solved base case AC power flow solution.
- (b) Specify the bootstrap sample, B , and the percentage of bootstrap confidence interval specified by α . Obtain the large uncertainty of ATCs estimated at a certain percentage of bootstrap confidence interval, $ATC_{n,b=q1}$, by using the procedure described in Section 2.1. The procedure of bootstrap technique is performed repeatedly in order to obtain the bootstrap sample of $ATC_{n,b=q1}$ at various factors or system operating conditions. The factors are such as the generation dispatch, customer demand, system parameters and system topology [21]. Therefore, $ATC_{n,b=q1}^g$ is representing as the bootstrap sample of ATCs at various factors and it is estimated at a certain percentage of bootstrap confidence interval. Where, $g = 1, 2, 3, \dots, G$ and G is the total number of factors considered in the analysis. In this case study, the inherent or actual variables of transmission line impedances and hourly peak loads in a day are the factors considered in the determination of $ATC_{n,b=q1}^g$ by using the bootstrap techniques.
- (c) Determine the parametric or non-parametric bootstrap samples estimated at 0% of bootstrap confidence interval, $ATC_{n,b=0\%}^g$. The samples represents as the inherent variables of ATC in a day at various factors.
- (d) For the specified area-to-area transfer, determine the TRM at n th time interval, TRM_n , by using (5).

$$TRM_n = \sqrt{\sum_{g=1}^G \left(ATC_{n,b=0\%}^g - ATC_{n,b=q1}^g \right)^2} \quad (5)$$

Equation (5) is the standard deviation formulation that used to determine the TRM which is basically based on the sum of variances of independent random variables. The TRM computation proposed by Zhang *et al.*[21] was basically based on the formulation of standard error or standard deviation. The standard deviation is often used to approximate confidence interval of a

standard normal distribution [11,12,22]. Nevertheless, the standard deviation has also been used to estimate the confidence interval of a distribution generated by the bootstrap technique [11,12,23]. This shows that the confidence interval of a distribution generated by the bootstrap technique can be estimated either by using the bootstrap confidence interval as described in Section 2.1 or by using the standard deviation formulation. In particular, given that the mean, μ , and standard deviation, σ , are known for a standard normal distribution. For instance, 90% confidence interval for a standard normal distribution is $\mu \pm 1.645 \times \sigma$. Whereby, 1.645 is the value obtained from the standard normal table and it is selected referring to the 90% of the confidence interval [11,12,22]. However, the $\mu \pm 1.645 \times \sigma$ can sometimes be quite inaccurate in estimating the confidence interval of a standard normal distribution [22]. Hence, the bootstrap confidence interval is normally used to determine accurate confidence interval of a standard normal distribution [22]. In general, $[\mu - Z^{(1-\alpha)} \times \sigma, \mu + Z^{(\alpha)} \times \sigma]$ is called as the standard confidence interval with the coverage probability equal to $1 - 2\alpha$ [11,12,22]. In the bootstrap technique, the $1 - 2\alpha$ percentile interval is defined by the α and $1 - \alpha$ percentiles of a standard normal distribution [22]. As explained in Section 2.1, $q_1 = (B\alpha/2)$ and $q_2 = B - q_1 + 1$ is used to select the range of confidence interval based on the desired $(1 - \alpha)$ 100%. The above-mentioned explanations show that the bootstrap confidence interval and standard deviation are used to estimate the confidence interval of a distribution in which it is the basis for TRM determination. This proves that the bootstrap confidence interval can also be used to determine TRM, in which it is relatively similar to the TRM value determined by the standard deviation.

The determination of TRM_n at each n th time interval is explained elaborately in the following procedures. Firstly, identify the value of $ATC_{n,b=0\%}^g$ at each n th time interval. This procedure is performed on the chronological of $ATC_{n,b=0\%}^g$. The chronological arrangement of $ATC_{n,b=0\%}^g$ is similar to the chronological arrangement of actual or inherent ATCs. Second, identify the percentile of variability for $ATC_{n,b=0\%}^g$ at each time interval. The percentile of variability is also called as the percentage of normal CDF. This procedure is performed on the normal CDF of $ATC_{n,b=0\%}^g$. Third, for every factor or system operating condition, g ; determine the numerical difference between $ATC_{n,b=q_1}^g$ and $ATC_{n,b=0\%}^g$ at each percentile of variability. The differences between ATCs at each factor will be applied into (5). This procedure is performed on the normal CDFs of $ATC_{n,b=q_1}^g$ and $ATC_{n,b=0\%}^g$. Fourth, for every factor, allocate the difference between ATCs at each time interval referring to the percentile of variability identified in the second procedure. Fifth, for a particular time interval represented by the percentile of variability, obtain the summation on the squared difference between ATCs of all the factors. Sixth, the result obtained in the fifth procedure is applied into (5) in order to obtain the TRM at a particular time interval. Seventh, repeat the fifth and sixth procedures in order to obtain the TRM at every time interval, TRM_n .

On the other hand, considering the fact that ATC is an estimate of the future power transfer availability [10–12], therefore, it is important to determine TRM based on a future distribution of ATCs. In this analysis, the historic or inherent variables of ATC (ATC_n) are used as a guide to estimate future distribution of ATCs ($ATC_{n,b=q_1}$) for the TRM determination. The procedure of bootstrap technique is performed repeatedly in order to obtain the future distribution of ATCs at various factors or system operating conditions and it is represented by the $ATC_{n,b=q_1}^g$.

- (e) Repeat steps (b–d) in order to determine the TRM at each time interval for the subsequent cases of power transfer.

The TRM value at a particular time interval is selected based on a certain percentage of normal CDF. The percentage of normal CDF is also called as the percentile of variability. Variability represents diversity or heterogeneity in data, which is not reducible through further measurement or study [6–8]. On the other hand, uncertainty arises due to the limited amount of information that is used to characterize the entire data [6–8] and it is used in estimating the TRM. In particular, a large TRM value is obtained depending on the ATCs with large uncertainty, $ATC_{n,b=q_1}^g$, selected at a higher percentage of bootstrap confidence interval. This shows that the TRM value is varied by changing the percentage of bootstrap confidence interval and also the TRM at a particular time interval is obtained based on a certain percentile of variability.

3. DETERMINATION OF TRM CONSIDERING UNCERTAINTIES OF TRANSMISSION LINE OUTAGES AND SYSTEM OPERATING CONDITIONS

This section discusses on the methodology of TRM determination considering the line outages and system operating conditions. Initially, the bootstrap technique is used to provide several samples of transmission line outages. The ATC is then calculated for every sample of transmission line outages. The TRM is determined based on the standard deviation of ATC estimated at certain percentage of confidence interval. The following procedure presents the determination of TRM by considering the uncertainty of transmission line outages up to $N-2$.

- (a) Specify a sample of transmission lines status, S , consisting of two line outages, 0, and the rest is in-service, 1. The outage of two transmission lines is selected randomly.
- (b) Use a random number generator to draw a random sample of m values, and replace the actual sample of S_m with the non-parametric bootstrap sample of S_m^* .
- (c) Repeat step (b) in order to obtain non-parametric bootstrap samples of $S_{m,b}^*$.
- (d) Discard any sample comprising of more than two transmission line outages. Retain any sample with one, $(N-1)$, and two, $(N-2)$, transmission line outages, and proceed to the next procedure.
- (e) Use an AC power flow solution to determine the ATC for every non-parametric bootstrap sample of $S_{m,b}^*$. The determination of ATC based AC power flow solution is explained elaborately in Section 4.
- (f) Sort the ATC_b in ascending order.
- (g) Determine the TRM by using Equation (6). In the Equation (6), the TRM_{outage} at a certain percentage of normal probability density function (%PDF) is obtained based on the standard deviation of ATCs.

$$TRM_{outage} = D\sigma_{ATC} \tag{6}$$

In Equation (6), the TRM_{outage} at a certain percentage of %PDF is obtained by multiplying D with the standard deviation of ATC (σ_{ATC}). D is a constant that used to increase the value of TRM as it is set to a higher percentage of normal probability density function (%PDF). The value of D that refers to the %PDF [24,25] is determined from

$$\%PDF = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu-(D\sigma)}^{\mu+(D\sigma)} e^{\left(\frac{-(t-\mu)^2}{2\sigma^2}\right)} dt \times 100\% \tag{7}$$

Therefore, 68, 95 and 99.7% of %PDF gives D values of 1, 2 and 3 [24,25], respectively.

In a parametric bootstrap technique, the TRM_{outage} in Equation (6) is combined with Equation (5) at every n th time interval in order to calculate a new TRM (TRM_n^{new}) that takes into account the uncertainties of transmission line outages and system operating conditions. This shows that the TRM_{outage} in Equation (6) represents as one of the factors, g , for Equation (5). In this case, the same percentages of %PDF and bootstrap confidence interval give relatively similar amount of uncertainty. Therefore, the (TRM_n^{new}) selected at a certain percentage of confidence interval can be obtained by using Equation (8)

$$TRM_n^{new} = \sqrt{TRM_n^2 + TRM_{outage}^2} \tag{8}$$

By applying the TRM_n into Equation (8) therefore, a new TRM (TRM_n^{new}) is be obtained.

4. ATC ASSESSMENT CONSIDERING TRM

In general, the procedure used in determining the ATC involves the definition of a base case, determination of network response and finding the maximum power transfer or TTC. In this analysis, ATC of a transmission system is determined by performing recursive AC power flow solution under

specific set of operating conditions. Determination of interarea ATC using recursive AC power flow solution is described in the following procedure:

- (a) Establish a solved base case AC power flow solution.
- (b) Specify the areas of power transfer. The area-to-area transfer considers participation of all generators in the specified selling area and all loads in the specified buying area.
- (c) Simultaneously, increase equal increments of power injection and extraction at both sides of the selected areas until either one of the MVA power flow limit or voltage magnitude limit is reached.
- (d) Calculate the ATC that is given by the maximum power transfer at the limiting case (or TTC), less the base case power flow and less the TRM.

In step (d), the actual ATC value at each time interval can be obtained without considering the TRM. On the other hand, Equation (1) requires the value of base case ATC (ATC^0) in which it is obtained by considering the base case system operating conditions and also without considering the TRM.

5. RESULTS AND DISCUSSION

The effectiveness of the proposed TRM method is validated on a simplified 241 bus Malaysian power system. The 241 bus Malaysian power system is comprised of five areas, namely, area North, area East, area Central, area South and area PUB. It has 143 generation units, 98 load units and 368 transmission lines. The total generation and peak load are 11 919 and 11 866 MW, respectively. The system operating condition that is considered in this analysis is the inherent or actual variables of transmission line impedances, hourly peak loads in a day and transmission line outages.

5.1. Estimation of TRM using parametric bootstrap technique

In the parametric bootstrap technique, selecting an appropriate parametric distribution of variable inputs is an important task because it affects the integrity of uncertainty results. The significance of selecting an appropriate parametric distribution is to ensure that the type of parametric distribution provides generated variables that consist of similar distribution as the inherent variables. Without selecting an appropriate parametric distribution it may lead to an ambiguous result of uncertainty. Four types of parametric distribution which are the normal, lognormal, Poisson and Rayleigh CDFs considered for comparing it with the empirical distribution function of inherent variables. The empirical distribution function is considered as the actual parametric distribution for the inherent or actual variables of ATC in a day. In this analysis, the parametric distribution of inherent ATCs is developed based on the inherent variables of Malaysian hourly peak load which are selected on the 1st March 2002. The inherent ATCs are determined based on the transfer case from area North to area East. The result of four types of parametric distribution is shown in Figure 1. The basic statistical formulations provided by the MATLAB are used to compute the empirical distribution function and the four parametric distributions. By comparing the result of the parametric distributions, the normal CDF offers the best fit to the empirical distribution function. The result shows that the normal CDF used in the parametric bootstrap technique is suitable in providing accurate samples of ATCs in a day with large uncertainty.

Based on the selected type of fitted parametric distribution, the parametric bootstrap technique uses the mean and variance of the normal CDF to generate several samples of ATCs in a day with large uncertainty. In the parametric bootstrap technique, the number of bootstrap samples ranging between $B = 1000$ and 2000 is suggested so as to provide an accurate estimation of uncertainty [6]. In the analysis, the bootstrap samples of $B = 2000$ is chosen and each parametric bootstrap sample represents the ATCs in a day with large uncertainty. The criteria for each parametric bootstrap sample can be described in terms of normal CDF. Hence, an abscissa of normal CDF is constructed based upon the 2000 parametric bootstrap samples, $ATC_{n,b}^g$, as shown in Figures 2 and 3. The basic statistical formulation provided by the MATLAB is used to compute the normal CDF of each parametric bootstrap sample in order to obtain the abscissa as depicted in Figures 2 and 3. The abscissa of normal

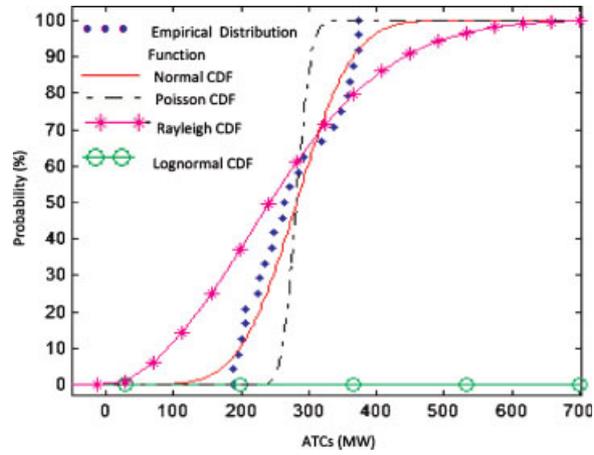


Figure 1. Comparison of parametric distributions that fit to the empirical distribution function.

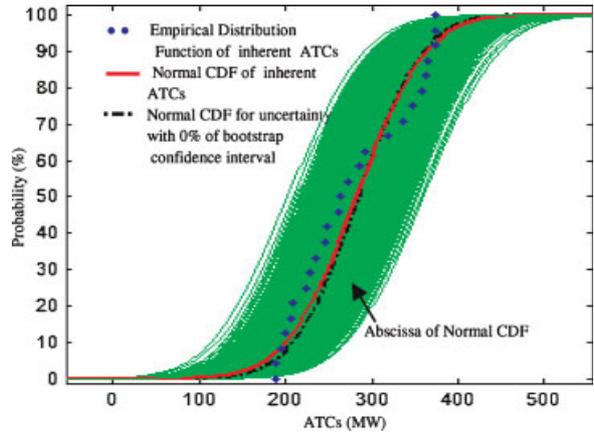


Figure 2. Normal CDFs of inherent ATCs and 0% uncertainty with empirical distribution function located at the centre of abscissa.

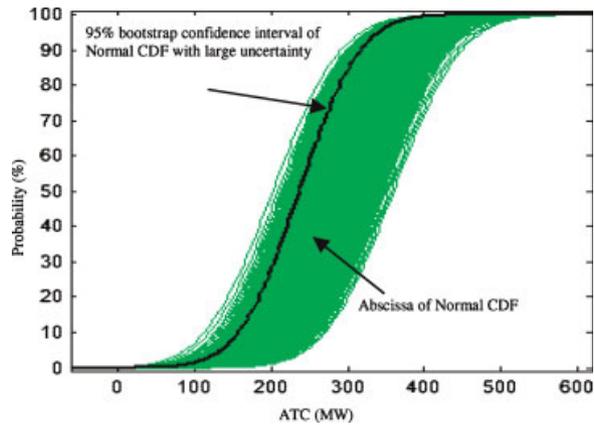


Figure 3. Normal CDF of the ATCs with large uncertainty selected at the 95% of bootstrap confidence interval.

CDF in Figure 2 shows that several numbers of normal CDF are located beyond the range of empirical distribution function. This implies that the parametric bootstrap technique is capable in providing realistic information of large uncertainty that refers to the generated ATCs in a day which are not confined within the maximum and minimum range of inherent variables of ATC, and this may occur in the future ATCs in a day. Whereby, the empirical distribution function represents as the actual parametric distribution of ATCs in a day. It can be seen in Figure 2 that normal CDF for the inherent ATCs is similar to the normal CDF of ATCs with uncertainty at 0% of bootstrap confidence interval. Consequently, both the parametric distributions are located at the centre of abscissa. Hence, it is proven that the parametric bootstrap technique is robust in providing accurate estimation of uncertainty due to the fact that the 0% of bootstrap confidence interval provides similar normal CDF as the inherent variables of ATC in a day.

As the percentage of bootstrap confidence interval increases, the skewness of normal CDF will increase due to the increase of large uncertainty in ATCs. This makes the normal CDF of ATCs with large uncertainty is not symmetrical to the centre of abscissa. Parametric bootstrap technique with a higher percentage of bootstrap confidence interval will produce a large amount of uncertainty in the ATCs in a day. In this example, the large uncertainty of ATCs is chosen at a bootstrap confidence interval of 95%, $ATC_{n,b=95\%}^g$. The 95% bootstrap confidence interval of ATCs is illustrated in terms of normal CDF as shown in Figure 3. The basic statistical formulation provided by the MATLAB is used to compute the normal CDF of ATCs selected at the 95% of bootstrap confidence interval. It is noted that the normal CDF of ATCs with large uncertainty is negatively skewed towards the left side of the abscissa.

It is worth noting that the parametric bootstrap sample is generated by the parametric bootstrap technique that takes into account the inherent variable of ATCs in a day. The inherent variable of ATCs in a day is referring to the transfer case from area North to area East. The inherent variable of ATCs in a day is obtained by using (1) that takes into account the hourly peak loads selected on the 1st March 2002. Then, the parametric bootstrap sample of ATCs was selected at the 95% of bootstrap confidence interval, $ATC_{n,b=95\%}^g$. Subsequently, comparative study is made between the normal CDFs of ATCs which are obtained based on the parametric bootstrap sample and the hourly peak loads of 5th June 2002. The normal CDF for both samples are relatively similar and it is shown in Figure 4. This proves that the parametric bootstrap technique is robust in providing accurate distribution of ATCs in a day which reflects the future ATCs in a day. This is important for the TRM determination because it considers accurate estimation of uncertainty in transfer capability which may cause the value of ATCs in present day is similar to the value of ATCs in future.

The main concept of TRM determination using the parametric bootstrap technique is explained in the following case study. The concept is initially referring to the parametric bootstrap sample of ATCs generated at the 95% bootstrap confidence interval, $ATC_{n,b=95\%}^g$. The parametric bootstrap sample of ATCs that is the $ATC_{n,b=95\%}^g$ is referring to the transfer case from area North to area East.

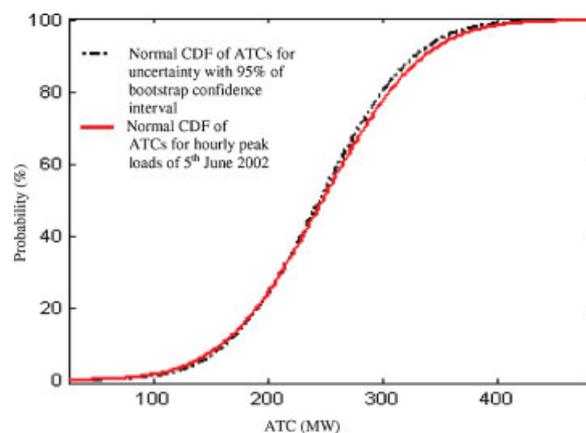


Figure 4. Normal CDFs of ATCs for uncertainty with 95% of bootstrap confidence interval and the hourly peak loads of 5th June 2002.

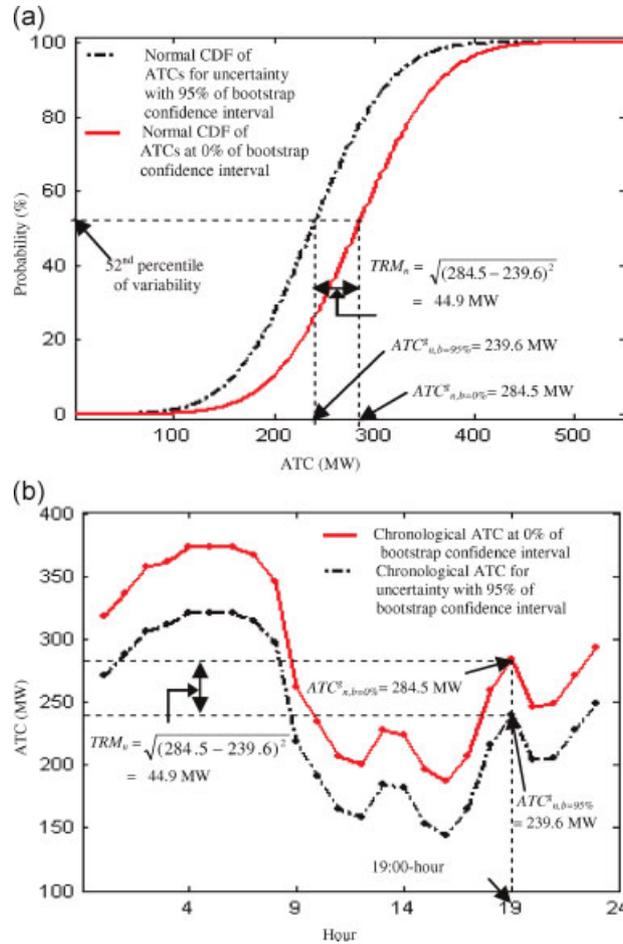


Figure 5. (a) Normal CDFs of ATCs for uncertainty with 95 and 0% of bootstrap confidence intervals. (b): Chronological ATCs for uncertainty with 95 and 0% of bootstrap confidence intervals.

Simultaneously, the normal CDF of $ATC_{n,b=95\%}^g$ is obtained and it is shown in Figure 5a. The normal CDF of ATCs at 0% of parametric bootstrap sample, $ATC_{n,b=0\%}^g$, is also depicted in Figure 5a. The normal CDF of $ATC_{n,b=0\%}^g$ is relatively similar to the normal CDF of inherent ATCs that refers to the inherent hourly peak loads of 1st March 2002. Then, the TRM at each time interval, TRM_n , is obtained by using (5) in which it is based on the values of $ATC_{n,b=95\%}^g$ and $ATC_{n,b=0\%}^g$. Figure 5a shows that a certain percentile of variability represents as a particular time interval of TRM (TRM_n). For instance, the 52nd percentile of variability provides the TRM value of 44.9 MW at 19:00. On the other hand, the $ATC_{n,b=95\%}^g$ and $ATC_{n,b=0\%}^g$ are chronologically depicted in Figure 5b. The $ATC_{n,b=95\%}^g$ and $ATC_{n,b=0\%}^g$ are applied into (5) which gives the TRM value at every time interval and the TRM of 44.9 MW at 19:00 is shown in Figure 5b. Furthermore, large TRM value at every time interval can be obtained by increasing the percentage of bootstrap confidence interval.

The TRMs obtained on the 1st March 2002 are then used to determine new ATCs on the 2nd March 2002. The new ATCs on the 2nd March 2002 are obtained by referring to the actual ATC values on the 2nd March 2002 less the TRMs obtained on the 1st March 2002 and this is illustrated in Figure 6. In Figure 6, 255.6 MW is the value of new ATC that is obtained due to the actual ATC value of 300.5 MW less the TRM value of 44.9 MW and this is referring to the time interval of 19:00. The TRM value of 44.9 MW is obtained initially at 19:00 on the 1st March 2002 and this is shown in Figure 5b. The new and actual ATCs are obtained at 19:00 on the 2nd March 2002 and it is illustrated in Figure 6.

Consecutively, the non-parametric bootstrap technique is performed to obtain the outages of transmission line for TRM_{outage} computation. This technique is chosen due to its ability to randomly

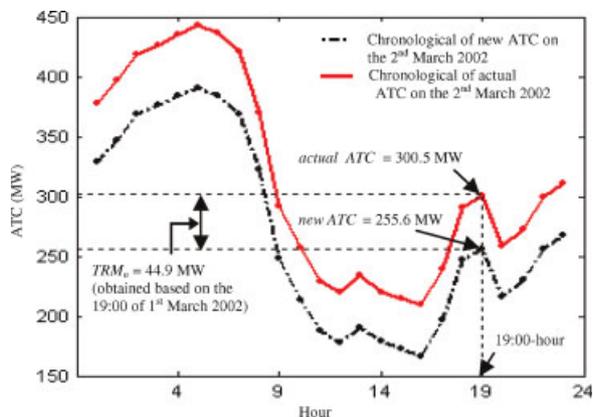


Figure 6. Chronological of new and actual ATCs on the 2nd March 2002.

allocate one, $N-1$, or two, $N-2$, outages of transmission line for every non-parametric sample. In particular, the outage of a transmission line is specified as '0' and '1' is representing as the operating of a transmission line. The $N-2$ state represents as simultaneous outages of two transmission line and the outage of one transmission line is defined as state $N-1$. The states with three or more simultaneous outage transmission lines are not taken into consideration in this case study. This is because the probability of occurrence is small for three or more simultaneous transmission line outages. The ATC is then calculated for every non-parametric sample of transmission line outages. The interarea transfer from area North to area East is still considered in this case study. The TRM_{outage} is determined based on standard deviation of ATC estimated at a certain percentage of confidence interval. The TRM_{outage} is then used in Equation (8) in order to determine the TRM_n^{new} value at every n th time interval. Since, Equation (6) is basically a standard deviation formulation that is relatively equivalent to Equation (5). Therefore, TRM_{outage} can be considered as one of the factors, g , for TRM_n^{new} calculation. The combined uncertainties of transmission line outages and hourly peak load give larger value of TRM_n^{new} . This can be observed by comparing between the TRM results shown in Figure 7 and Figure 5b. For instance, by referring to the 52nd percentile of variability shown in Figure 7 and Figure 5b, the combined uncertainties give the TRM_n^{new} value of 62.14 MW that is larger than the TRM_n value of 44.9 MW obtained by only considering the uncertainty of hourly peak load, respectively. It is worth mentioning that a certain percentile of variability is representing as a particular time interval of TRM_n^{new} . Thus, the 52nd percentile is equivalent to the $n = 19:00$ of time interval. Simultaneously, the combined uncertainty of transmission line outages and hourly peak load further reduce the amount of

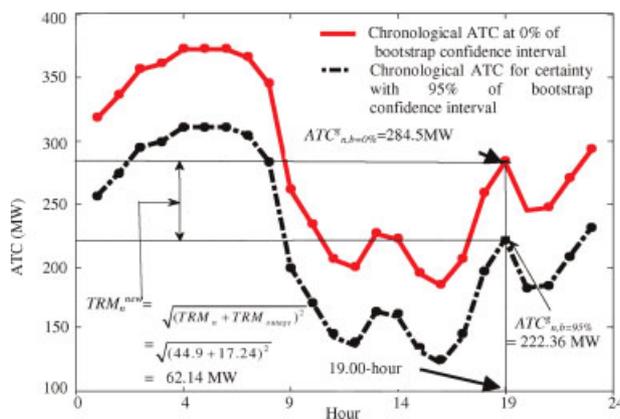


Figure 7. Chronological ATCs for uncertainty with 95 and 0% of bootstrap confidence intervals considering the combination of uncertainties of transmission line outages and hourly peak load.

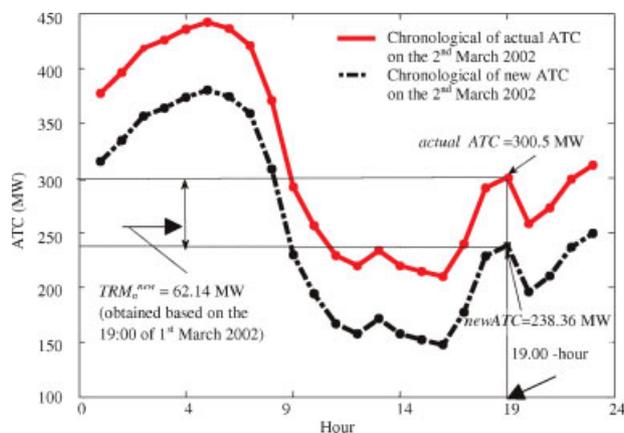


Figure 8. Chronological of new and actual ATCs on the 2nd March 2002 considering the combined uncertainties of transmission line outages and hourly peak load.

ATC. This is referring to the $ATC = 300.5$ MW at 19:00 on the March 1, 2002 shown in Figure 8. The new $ATC = 238.36$ MW is obtained by referring to the actual $ATC = 300.5$ MW less the TRM_n^{new} value of 62.14 MW.

In general, comparative study between Figures 8 and 6 proves that the combined uncertainties of transmission line outages and hourly peak load significantly increases the value of TRM_n^{new} yielding to a lower value of new ATC as compared by only considering the uncertainty of hourly peak load. Figures 7 and 5b represent the chronological ATCs on March 1, 2002. The TRMs obtained on March 1, 2002 are used to determine new ATCs on March 2, 2002. The new ATCs on March 2, 2002 are obtained by referring to the actual ATC values on March 2, 2002 less the TRMs obtained on March 1, 2002 and this is depicted in Figure 8. By referring to the time interval of 19:00, the TRM_n^{new} value of 62.14 MW reduces the actual ATC of 300.5 MW to a new ATC value of 238.36 MW. This shows that the combined uncertainties of transmission line outages and hourly peak load significantly reduces the new ATC value of 238.36 MW that is lower than the new ATC value at 255.6 MW shown in Figure 6. In Figure 6, the new ATC value of 255.6 MW is obtained only due to the impact of uncertain hourly peak occurred at 19:00 on March 2, 2002.

5.2. Results of TRM and ATC

Dynamic changes in transfer capability at every time interval require the use of probabilistic approach to determine the TRM. In this analysis, comparisons are made on the results of TRM which are obtained based on the parametric and non-parametric bootstrap techniques. Furthermore, comparisons in terms of accuracy and total computational time in determining the TRM are made between the two bootstrap techniques. The obtained TRM value is then used in a new ATC determination. The results of TRM and new ATC are presented based on several percentages of bootstrap confidence interval.

The TRM and new ATC for the Malaysian power system are determined by referring to the transfer cases from area North to area East and also from area South to area PUB. In this case study, the actual ATC for transfer cases from area North to area East and from area South to area PUB are found to be 317.72 and 137.51 MW, respectively. The actual values of ATC are obtained at the first hour or 0:00 on the 2nd March 2002. The TRM at 0:00 is determined based on the ATCs that take into account the inherent variables of transmission line impedances and hourly peak loads which are selected on the 1st March 2002. Table I shows the results of TRM at 0:00 on the 1st March 2002 and it is referring to the transfer case from area North to area East. On the other hand, Table II shows the results of TRM at 0:00 on the 1st March 2002 and it is obtained based on the transfer case from area South to area PUB. It is shown in Tables I and II that the TRM increases as the uncertainty is selected at higher percentage of bootstrap confidence interval.

Table I. Results of TRM at 0:00 for the transfer case from area North to area East considering the parametric and non-parametric bootstrap techniques.

Parametric bootstrap technique			Non-parametric bootstrap technique		
Bootstrap confidence interval (%)	TRM (MW)	CPU time (minutes)	Bootstrap confidence interval (%)	TRM (MW)	CPU time (minutes)
0	0	5.15	0	0	6.45
10	3.63		10	2.47	
20	7.54		20	5.85	
30	10.41		30	7.63	
40	14.77		40	9.87	
50	16.23		50	12.98	
60	21.23		60	14.78	
70	26.56		70	18.53	
80	31.64		80	23.62	
90	41.21		90	29.06	
95	48.01		95	34.03	
99	64.67		99	44.39	
99.5	67.09		99.5	49.87	

Table II. Results of TRM at 0:00 for the transfer case from area South to area PUB considering the parametric and non-parametric bootstrap techniques.

Parametric bootstrap technique			Non-parametric bootstrap technique		
Bootstrap confidence interval (%)	TRM (MW)	CPU time (minutes)	Bootstrap confidence interval (%)	TRM (MW)	CPU time (minutes)
0	0	4.87	0	0	6.06
10	2.09		10	1.71	
20	3.93		20	2.97	
30	5.99		30	4.87	
40	8.01		40	6.07	
50	11.13		50	8.37	
60	13.85		60	10.22	
70	16.12		70	12.76	
80	20.62		80	15.91	
90	24.07		90	19.12	
95	30.56		95	22.98	
99	39.45		99	29.61	
99.5	44.74		99.5	33.37	

Comparisons are made in terms of accuracy and total computational time to distinguish the effectiveness of the two methods in determining the TRM. The compared methods are the parametric and non-parametric bootstrap techniques. In Tables I and II, it is noted that the parametric bootstrap technique provides higher value of TRM as compared to the non-parametric bootstrap technique. This is due to the fact that the non-parametric bootstrap technique provides the uncertainty which is confined between the minimum and maximum values of observed data. This implies that the parametric bootstrap technique has the advantage of providing realistic information of uncertainty as compared to the non-parametric bootstrap technique.

In Table I, the TRMs are determined by the parametric and non-parametric bootstrap techniques with the total computational time of 5.15 and 6.45 minutes, respectively. In Table II, the parametric and non-parametric bootstrap techniques estimate the TRM values with total computational time of 4.87 and 6.06 minutes, respectively. This shows that the resampling process in the bootstrap techniques

Table III. Results of new ATC at 0:00 for the transfer case from area North to area East considering the parametric and non-parametric bootstrap techniques.

Bootstrap confidence interval (%)	ATC (MW) considering parametric bootstrap technique	ATC (MW) considering non-parametric bootstrap technique
0	317.72	317.72
10	314.09	315.25
20	310.18	311.87
30	307.31	310.09
40	302.95	307.85
50	301.49	304.74
60	296.49	302.94
70	291.16	299.19
80	286.08	294.10
90	276.51	288.66
95	269.71	283.69
99	253.05	273.33
99.5	250.63	267.85

yield to a lengthy computational time in estimating the TRMs. Although it is time consuming, both the methods have the advantage of providing simultaneous TRM value at all confidence intervals of bootstrap technique and this is important to ensure that the transmission network is secure from various large uncertainties that may occur during a power transfer. Furthermore, the advance development of computer technology may able to overcome the drawback in terms of time consuming in computing the TRM.

Tables III and IV are the results of new ATC at 0:00 on the 2nd March 2002 which are obtained referring to the actual ATCs less the TRMs given in Tables I and II, respectively. The transfer cases from area North to area East and from area South to area PUB give the actual ATC values of 317.72 and 137.51 MW, respectively. The actual values of ATC are obtained at the first hour or 0:00 on the 2nd March 2002. Tables III and IV prove that the parametric bootstrap technique provides lower values of new ATC than the non-parametric bootstrap technique. The small values of new ATC are obtained due to the TRM with large uncertainty predicted beyond the range of inherent variables. On the other hand, the results of new ATC decreases as the TRM is increased by the percentage of bootstrap confidence interval.

The same case study as above is used to determine the TRM that takes into account the combined uncertainties of transmission line outages, transmission line impedances and hourly peak loads. Table V represents the result of TRM at 0:00 on March 1, 2002 and it is based on the transfer case from

Table IV. Results of new ATC at 0:00 for the transfer case from area South to area PUB considering the parametric and non-parametric bootstrap techniques.

Bootstrap confidence interval (%)	ATC (MW) considering parametric bootstrap technique	ATC (MW) considering non-parametric bootstrap technique
0	137.51	137.51
10	135.42	135.80
20	133.58	134.54
30	131.52	132.64
40	129.50	131.44
50	126.38	129.14
60	123.66	127.29
70	121.39	124.75
80	116.89	121.60
90	113.44	118.39
95	106.95	114.53
99	98.06	107.90
99.5	92.77	104.14

Table V. Results of TRM at 0:00 for the transfer case from area North to area East considering the parametric and non-parametric bootstrap techniques.

Parametric bootstrap technique			Non-parametric bootstrap technique		
Bootstrap confidence interval (%)	TRM (MW)	CPU time (minutes)	Bootstrap confidence interval (%)	TRM (MW)	CPU time (minutes)
0	0	5.15	0	0	6.45
10	4.93		10	3.72	
20	10.25		20	8.81	
30	14.15		30	11.50	
40	20.07		40	14.87	
50	22.06		50	19.56	
60	28.85		60	22.27	
70	36.10		70	27.92	
80	43.00		80	35.59	
90	56.00		90	43.78	
95	65.25		95	51.27	
99	87.89		99	66.88	
99.5	91.18		99.5	75.13	

area North to area East. On the other sides, Table VI is the results of TRM at 0:00 on March 1, 2002 and it is obtained based on the transfer case from area South to area PUB. Tables V and VI again show that the TRM increases as the uncertainty is selected at higher percentage of bootstrap confidence interval. By comparing between the TRM results shown in Tables I and V, it is obvious that the combined uncertainties of transmission line outages, transmission line impedances and hourly peak loads yield to the TRM values that are larger than the TRMs which are obtained by only considering the transmission line impedances and hourly peak loads, respectively. This is similar to the comparison between the results of TRM shown in Tables II and VI. The TRM results are referring to the transfer case from area South to area PUB at 0:00 on March 1, 2002.

Tables VII and VIII represents the results of new ATC at 0:00 on 2 March 2002. The new ATCs shown in Tables VII and VIII are obtained by referring to the actual ATCs less the TRM values given in Tables V and VI, respectively. In this case study, the actual ATC for transfer cases from area North to area East and from area South to area PUB are found to be 317.72 and 137.51 MW, respectively. It is

Table VI. Results of TRM at 0:00 for the transfer case from area South to area PUB considering the parametric and non-parametric bootstrap techniques.

Parametric bootstrap technique			Non-parametric bootstrap technique		
Bootstrap confidence interval (%)	TRM (MW)	CPU time (minutes)	Bootstrap confidence interval (%)	TRM (MW)	CPU time (minutes)
0	0	4.87	0	0	6.06
10	3.27		10	2.30	
20	6.15		20	5.20	
30	9.37		30	8.52	
40	12.53		40	10.62	
50	17.41		50	14.65	
60	21.66		60	17.89	
70	25.21		70	22.33	
80	32.25		80	27.85	
90	37.65		90	33.46	
95	47.80		95	40.22	
99	61.71		99	51.82	
99.5	69.98		99.5	58.40	

Table VII. Results of new ATC at 0:00 for the transfer case from area North to area East considering the parametric and non-parametric bootstrap techniques.

Bootstrap confidence interval (%)	ATC (MW) considering parametric bootstrap technique	ATC (MW) considering non-parametric bootstrap technique
0	317.72	317.72
10	312.79	314.00
20	307.47	308.91
30	303.57	306.22
40	297.65	302.85
50	295.66	298.16
60	288.87	295.45
70	281.62	289.80
80	274.72	282.13
90	261.72	273.94
95	252.47	266.45
99	229.83	250.84
99.5	226.54	242.59

Table VIII. Results of new ATC at 0:00 for the transfer case from area South to area PUB considering the parametric and non-parametric bootstrap techniques.

Bootstrap confidence interval (%)	ATC (MW) considering parametric bootstrap technique	ATC (MW) considering non-parametric bootstrap technique
0	137.51	137.51
10	134.24	135.21
20	131.36	132.31
30	128.14	128.99
40	124.98	126.89
50	120.10	122.86
60	115.85	119.62
70	112.30	115.18
80	105.26	109.66
90	99.86	104.05
95	89.71	97.29
99	75.80	85.69
99.5	67.53	79.11

worth mentioning that the TRMs are obtained by referring to the uncertainties of transmission line outages, transmission line impedances and hourly peak loads. By comparing between the new ATC results shown in Tables III and VII, it is observed that the combined uncertainties of transmission line outages, transmission line impedances and hourly peak loads yield to the new ATC values that are lower than the new ATC which are calculated by only considering the transmission line impedances and hourly peak loads, respectively. This discussion is similar to the comparison between the results of new ATC shown in Tables IV and VIII.

6. CONCLUSION

The need for efficiency in electric power deregulation has increased the needs to improve ATC calculation that takes into account the TRM. The importance of TRM is to ensure a secure operation of the interconnected network due to the impact of large uncertainty in transfer capability. This paper discussed on a new method to compute the TRM at each time interval for system operating condition and transmission line outage using the parametric bootstrap technique. The large uncertainty which is considered in the TRM value is referred to as the large uncertainty of ATCs provided by the parametric

bootstrap technique. Besides providing accurate estimation of large uncertainty, the parametric bootstrap technique has the advantage of selecting the large uncertainty based on the percentage of bootstrap confidence interval. The results have shown that the large value of TRM increases as the uncertainty provided by the parametric bootstrap is selected at higher percentage of bootstrap confidence intervals. On the other hand, it is obvious that the combined uncertainties of transmission line outages, transmission line impedances and hourly peak loads give the TRM values that are larger than the TRMs which are obtained by only considering the transmission line impedance and hourly peak load. The combined uncertainties of system operating condition and transmission line outages is important to be considered in the TRM determination in order to ensure efficient and secure operation of the interconnected system during power transfer. A comparative study has shown that the parametric bootstrap technique provides realistic information of uncertainty that yield to a higher value of TRM as compared to the non-parametric bootstrap technique.

7. LIST OF SYMBOLS

n	1, 2, 3, . . . , N
D	a constant specified by the percentage of confidence interval
ATC^o	base case ATC value of the specified area-to-area transfer case
x^o	base case system operating condition such as the rated load demand
$\partial ATC/\partial x$	the first order sensitivity which is the change of ATC with respect to the change of system operating condition such as the peak load
N	the total number of time intervals in a day
$rand_n$	random generated variables
σ_{ATC}	standard deviation of ATC obtained in step (e)
x_n	variables of system operating such as the hourly peak loads in a day

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